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Current risk-free interest rate

Cumulative normal distribution function

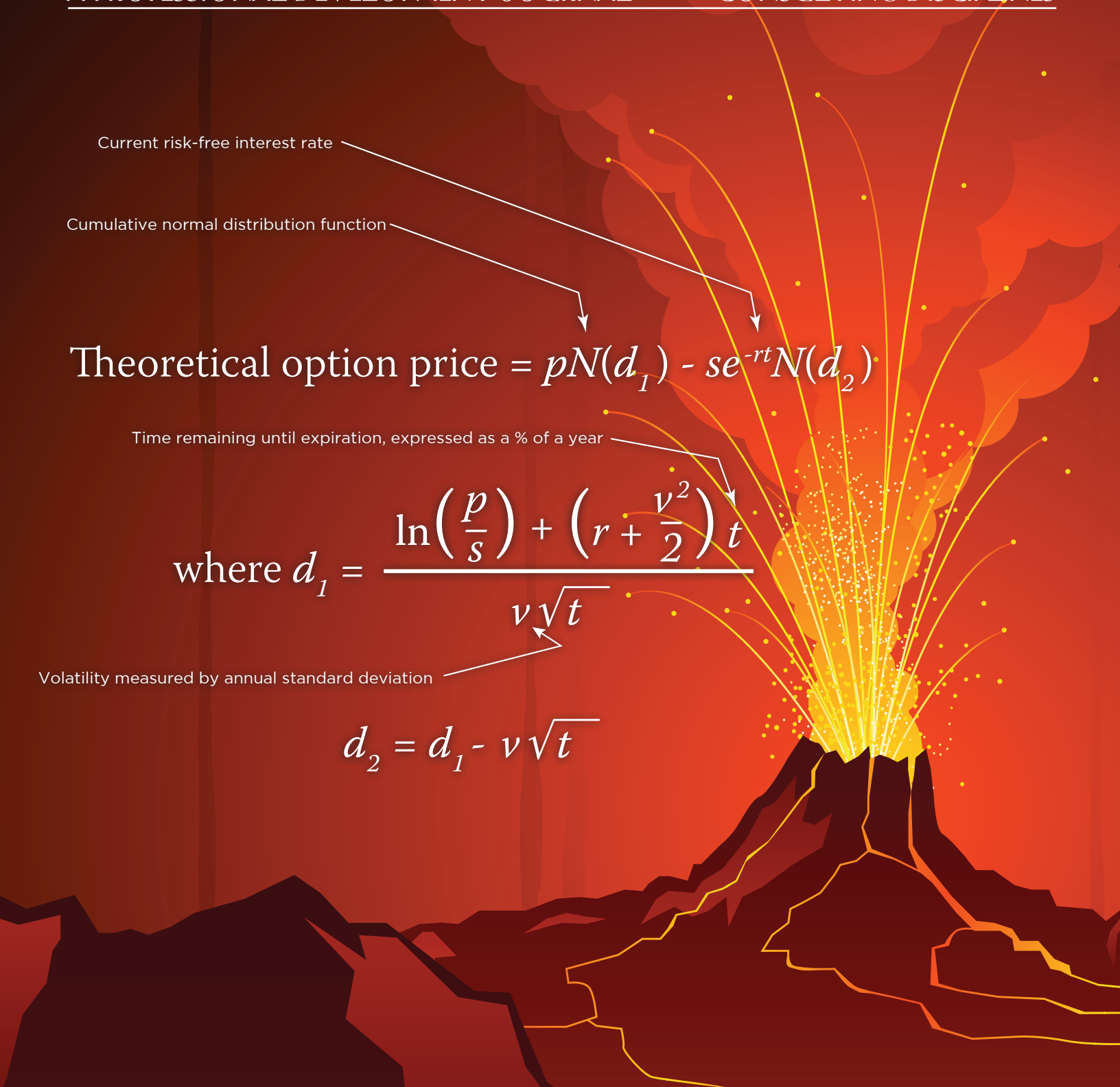
$$\text{Theoretical option price} = pN(d_1) - se^{-rt}N(d_2)$$

Time remaining until expiration, expressed as a % of a year

$$\text{where } d_1 = \frac{\ln\left(\frac{p}{s}\right) + \left(r + \frac{v^2}{2}\right)t}{v\sqrt{t}}$$

Volatility measured by annual standard deviation

$$d_2 = d_1 - v\sqrt{t}$$



VALUATION

Estimating Stock Price Volatility in the Black-Scholes-Merton Model

By Aaron M. Rotkowski, CFA, ASA

Closely held companies that issue stock options need to know the fair market value of these stock options for a variety of purposes, including dispute resolution, gift and estate tax compliance, transaction-related, reporting requirements, and others. This article presents analytical procedures that can be used to estimate the fair market value of stock options

- payment in a transaction
- When options are part of an estate that is subject to estate tax
- The divorce of an executive who holds options

issued by companies whose shares are not listed on a stock exchange or quoted on the NASDAQ.

The Black-Scholes-Merton (BSM) option pricing model is perhaps the most widely used option pricing model used by valuation analysts to estimate the fair market value of non-traded stock options issued by closely held companies. I will focus on the stock price volatility component of the BSM model. Stock price volatility is an important factor in many option valuation models—usually the most important factor. This particular input is difficult to estimate, is subject to the judgment of the valuator, and exerts a significant effect on the fair market value of the subject stock option.

The BSM option pricing model's popularity and acceptance have increased since it was first introduced in the early 1970s. It is noteworthy that in 1997 the BSM model authors won the Nobel Prize for their work in developing the option pricing model.

In this article I will not address share-based compensation for financial statement reporting purposes. For financial statement reporting purposes,

share-based compensation is regulated by FASB's Accounting Standards Codification (ASC) 718, formerly SFAS No. 123R.

I will first present a list of common reasons why valuation analysts are asked to estimate the fair market value of closely held company stock options. Second, I will discuss analytical procedures that can be used to estimate stock price volatility for closely held company stock.

REASONS TO VALUE STOCK OPTIONS

Employer companies and their advisers need to know the fair market value of stock options in a variety of situations, including (but not limited to) the following:

- Transferring ownership of options to a third party
- Damage suits in which the value of options is at issue, such as a breach of contract suit between an existing or former executive and the issuing company
- The repurchase of options by the issuing company
- When options are issued as a form of

ESTIMATING STOCK PRICE VOLATILITY

The five basic components of the BSM option pricing model are: (1) option time to expiration, (2) option exercise price, (3) risk-free interest rate, (4) current price of the underlying stock, and (5) expected volatility of the stock price over the life of the stock option.

The first three items listed are typically known with certainty; those values are not controversial in a dispute. However, when the company that issues the stock option is closely held, estimating the fourth and fifth items, stock price and stock volatility, require judgment by the analyst.

Estimating the stock price of a closely held company is a complicated issue. For purposes of this discussion, we shall assume that the stock price of the subject closely held company is known as of the option valuation date. Therefore, this discussion focuses on the stock price volatility estimate in the BSM model.

As stock price volatility increases, the likelihood that a stock's performance will be extreme (either positive or negative) increases. Since the employee owns a call option, he or she would be expected to benefit from price increases, but would

TABLE 1: STOCK OPTION VALUE SENSITIVITY TO VOLATILITY

Option Characteristics						
Stock Price	\$10.00					
Exercise Price	\$12.00					
Risk-free Rate	0.30%					
Time To Expiration	1 Year					
Expected Volatility	20%	30%	40%	50%	60%	70%
Indicated Stock Option Value	0.22	0.55	0.93	1.32	1.72	2.12

be protected from the downside risk that could otherwise occur from holding the stock directly. This is because the employee has nothing to lose (assuming no consideration was exchanged for the option).

Table 1 shows the sensitivity of a hypothetical stock option to changes in the expected volatility of the stock price, using the BSM option pricing model.

As shown in Table 1, when expected volatility increases from 30 to 40 percent, the indicated stock option value increases from \$0.55 to \$0.93, or by 69 percent. As expected volatility increases, incremental volatility changes of 10 percent have less of an effect on the indicated option value. That is, an increase in expected volatility from 20 to 30 percent has a greater effect on the option value than an increase in expected volatility from 60 to 70 percent, all other factors equal. The example in Table 1 illustrates the importance of the expected volatility estimate in the BSM model.

The volatility component of the BSM option pricing model is a forward-looking measure. The most logical starting place to look for an estimate of future volatility is past volatility.¹ Historical stock price volatility is

a function of the past variability in the returns on the stock as measured by changes in the stock price. These historical data are generally available for publicly traded companies. However, when valuing the options of a closely held company, reliable historical prices typically are not available.

The remainder of this article will present analytical procedures and other factors that valuation analysts can consider to estimate the expected stock price volatility for a closely held company. The analytical procedures and factors discussed in this article are not intended to provide a set of rules that valuation analysts must follow to estimate volatility for every closely held company. Determining the appropriate procedures to estimate the expected stock price volatility for a closely held company (a) should be determined on a case-by-case basis and (b) requires substantial professional judgment.

Valuators often estimate the expected volatility for a closely held company stock price by analyzing the volatility of reasonably comparable publicly traded companies (PTCs). There are several ways to perform this type of guideline company analysis. I will present three methods to conduct a guideline PTC volatility analysis.

The first procedure in any guideline company analysis is to select a group of PTCs that are similar to the subject company. A rule of thumb for the number of guideline companies required for a meaningful comparison is between three and 10 companies. Of course, the comparability—in terms of line of business, size, geographic diversity, product line diversification, and other factors—of the selected guideline PTCs to the subject company affects how many guideline PTCs are required for a meaningful comparison.

If the subject closely held company stock was appraised using a market approach valuation method, the same companies that were used in the market approach valuation method can be used as the guideline PTCs to estimate volatility for the stock of the subject company.

After the valuation analyst selects a group of guideline PTCs that are sufficiently comparable to the subject company, the next step is to analyze the volatility of these companies.

The first method of estimating volatility is to review and analyze the volatility that each guideline PTC reports in its SEC Form 10-K. Publicly traded companies that grant employees some form of share-based compensation (e.g., stock appreciation rights) are required under ASC 718 to measure the cost of all share-based payment plans at fair value. The financial statement footnotes of PTCs that follow ASC 718 typically include a description of how the fair value of the company's share-based compensation was determined. If the company used the BSM option pricing model, then it would disclose the expected stock price volatility.²

² It is important to reiterate that this discussion focuses on estimating the fair market value of closely held company stock options; it is not intended to provide accounting or valuation advice for the valuation of stock options for financial reporting purposes.

¹ Don M. Chance, *Analysis of Derivatives for the CFA Program*, Baltimore, MD, 2003, pg. 226.

The methods that PTCs use to estimate volatility vary from firm to firm. These estimates may be (a) forward-looking, (b) based on historical trading prices, or (c) a combination of those two factors.

For example, the expected stock price volatility estimated by CDI Corp. “is based on the historical volatility of the company’s common stock over the previous five years.”³

Conversely, Manpower estimates expected volatility, “using a weighted average of daily historical volatility (weighted 75 percent) of our stock price over the past five years and implied volatility (weighted 25 percent) based upon exchange traded options for our common stock.”⁴

A second method to estimate expected volatility for a closely held company stock price is to calculate the actual historical volatility of the stock prices for the selected guideline PTCs. This volatility is backward-looking. Historical volatility can be estimated over any period the valuation analyst selects. It is often appropriate to calculate historical volatility based on the same term as the subject stock option. That is, if the subject stock option has two years to expiration, then the valuator may want to calculate historical stock price volatility for the selected guideline PTCs over the two years prior to the option valuation date.

Representative results from the two comparable company analyses discussed above are presented in Table 2. This table shows the volatility analysis for a subject company that provides staffing services to companies located in the western United States. Therefore, the selected guideline PTCs also provide staffing services.

³ CDI Corp., SEC Form 10-k for the fiscal year ended December 31, 2010, page 54.

⁴ Manpower, Inc., annual report for the fiscal year ended December 31, 2010, page 57.

A third method to analyze the expected volatility for a closely held company stock price is to analyze the implied volatility in the guideline PTCs publicly traded stock options.

Using this method, the valuator can set the option price in the BSM option pricing model equal to the market price of the publicly traded stock option and work backwards to find the implied volatility. The implied volatility can be estimated this way because all other inputs required by the BSM option pricing model—including the option value—are known. This implied volatility represents the volatility that option traders are using to price the publicly traded stock option. The process used to infer the implied volatility is an iterative process that involves trial and error.

After (a) selecting guideline PTCs, (b) collecting the self-reported stock price volatility for the selected guideline PTCs, (c) calculating the historical stock price volatility for the selected guideline PTCs, and (d) calculating the implied volatility of the guideline PTCs publicly traded stock options (if available), the valuator has a reasonable basis for estimating a level of volatility that is appropriate for the subject company and its underlying stock and stock options. This is arguably the most important step in the comparable company analysis. It is also the step that requires the most judgment on the part of the valuator.

FACTORS TO CONSIDER

The factors in the following list may be considered by the valuation analyst when estimating stock price volatility for a closely held company based on guideline PTC data. This list is not comprehensive, and not all of the factors listed will apply in every engagement. You should use your professional judgment

to estimate stock price volatility for a closely held company when using the BSM option pricing model.

- If the subject company was previously appraised (e.g., if the subject company had quarterly or annual valuations prepared for ESOP administration purposes), then you can compare the historical change in the company stock price with the historical change in the stock prices of the selected guideline PTCs.
- Analyze the range of guideline PTC stock price volatility. If there are outliers, you may attempt to understand why a company has a particularly high or particularly low stock price volatility relative to the other guideline PTCs.
- Review guideline PTC volatility for a period of five or 10 years prior to the valuation date. If there was a significant change (e.g., merger, significant transaction, or restructuring) regarding a company, you may be required to analyze the company or industry in greater depth to determine what caused the change.
- Stratify the selected guideline PTCs to produce a smaller group of companies that is very similar to the subject company. For example, if the subject company operates conventional grocery stores, and the guideline PTCs operate both conventional and “all natural” grocery stores, you may want to analyze both (a) the entire group of guideline PTCs and (b) a subset of that group that only includes conventional grocers.
- Analyze summary statistics from the reported volatilities. These summary statistics may include the high, the low, the median, and the average.
- Compare the guideline PTC historical volatility with the guideline

TABLE 2: BLACK-SCHOLES-MERTON OPTION PRICING MODEL

Self-Reported Stock Price Volatility for the Guideline Publicly Traded Companies (%)

COMPANY NAME	5-YEAR		10-YEAR		VOLATILITY [a]									
	MEAN	MEDIAN	MEAN	MEDIAN	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Barrett Business Services	60.7	61.5	59.8	60.0	58.8	61.9	61.5	NA	NA	59.0	61.0	62.0	58.0	56.0
CDI Corp.	44.3	40.5	40.5	37.5	52.5	55.5	40.5	36.0	37.0	35.0	34.0	38.0	36.0	40.0
Hudson Highland Group	57.7	58.0	62.1	59.0	NA	NA	58.0	60.0	55.0	55.0	55.0	65.0	73.5	75.0
Insperty	NA	NA	85.2	89.0	NA	NA	NA	NA	NA	89.0	90.0	92.0	86.0	69.0
Kelly Services	21.3	21.3	28.5	30.0	NA	NA	NA	NA	21.3	27.0	30.0	31.7	31.0	30.0
Kforce	73.6	73.0	58.9	50.0	NA	NA	75.4	73.0	72.5	50.0	50.0	50.0	50.0	50.0
Manpower Group	34.0	30.0	33.9	34.9	41.0	42.0	30.0	27.0	30.0	30.0	39.8	40.1	40.9	17.7
On Assignment	59.3	51.7	58.5	58.0	73.3	73.1	51.5	47.1	51.7	58.0	58.0	59.0	58.0	55.0
Robert Half International	38.5	38.5	49.2	50.2	NA	NA	NA	NA	38.5	47.1	49.4	50.9	55.5	54.0
SFN Group	63.2	50.0	59.9	53.5	99.0	78.0	50.0	40.0	49.0	52.0	55.0	59.0	68.0	49.0
TrueBlue	48.0	43.5	61.9	56.3	59.6	53.0	43.5	40.5	43.5	50.0	79.0	89.0	79.0	82.0

MINIMUM	21.3	21.3	28.5	30.0
MAXIMUM	73.6	73.0	85.2	89.0
AVERAGE	48.9	45.2	53.8	51.8
MEDIAN	48.0	43.5	58.7	51.8

[a] Based on the implied and historical volatility of the company's stock options, as provided in the company's SEC Forms 10-K.

PTC self-reported volatility (which generally is a forward-looking measure). You should seek to understand any material differences between the two measures.

- Consider the strengths and weaknesses of the guideline PTC self-reported volatility. Strengths are: the measure is often forward-looking, and it is estimated specifically for use in the BSM

option pricing model. Weaknesses are: little detail is known about how the volatility was estimated, and it is subject to management judgment.

- Consider the strengths and weaknesses of the guideline PTC historical volatility. Strengths are: this measure is objective and can be calculated for the same term as the subject option. The primary weak-

ness of this measure is that it is backward-looking.

- You may consider the operations and financial situation of the subject company and the selected guideline PTCs. For example, if the companies operate in different regions, have varying amounts of intangible assets, are capitalized with different amounts of debt, or are disparate in

Historical Stock Price Volatility Estimates for the Guideline Publicly Traded Companies (%)

COMPANY NAME	5-YEAR		10-YEAR		HISTORICAL VOLATILITY [b]									
	MEAN	MEDIAN	MEAN	MEDIAN	2010	2009	2008	2007	2006	2005	2004	2003	2002	2001
Barrett Business Services	46.6	41.6	49.8	47.0	37.6	41.6	67.0	39.9	47.0	61.6	53.9	NA	NA	NA
CDI Corp.	58.7	48.8	48.2	44.2	48.8	75.5	80.7	40.1	48.2	26.7	35.9	31.4	39.4	55.0
Hudson Highland Group	79.2	66.3	73.3	66.3	60.3	115.1	98.9	55.4	66.3	84.9	32.2	NA	NA	NA
Insperity	50.7	47.4	62.4	50.4	46.5	47.4	65.5	46.7	47.5	53.3	46.2	63.8	124.8	82.4
Kelly Services	48.7	55.9	42.3	38.2	55.9	69.0	61.4	34.4	23.0	23.9	27.7	34.3	42.0	51.3
Kforce	49.3	44.6	61.7	58.0	41.6	60.5	61.9	37.8	44.6	41.7	55.4	82.9	97.7	92.9
Manpower Group	42.1	35.2	37.5	33.5	35.2	54.6	63.8	29.6	27.0	26.4	24.5	31.8	43.5	38.5
On Assignment	64.7	50.7	62.6	54.8	50.7	103.4	74.2	48.8	46.4	48.5	50.6	59.0	79.1	65.5
Robert Half International	39.5	32.3	40.4	36.4	32.3	48.3	55.1	31.0	30.6	33.5	32.3	39.4	49.3	52.4
SFN Group	79.1	64.7	65.6	59.2	64.7	123.1	107.5	56.1	44.3	40.3	39.8	54.6	63.1	62.3
TrueBlue	56.0	52.7	56.9	50.5	52.7	70.4	65.4	48.4	43.0	40.3	40.4	42.8	74.5	91.6

MINIMUM	39.5	32.3	37.5	33.5
MAXIMUM	79.2	66.3	73.3	66.3
AVERAGE	56.8	49.9	55.1	49.2
MEDIAN	53.3	49.8	59.3	50.5

[b] Equals the one-year (260 trading days) historical volatility at calendar year-end, per Bloomberg.

Sources: SEC Forms 10-K, Bloomberg, and Willamette Management Associates calculations.

size, then the stock price volatility for the guideline PTCs may be greater than or less than what is appropriate for the subject stock option.

The analyses listed above relate primarily to comparisons between the subject company and the selected guideline PTCs. However, the valuator should also consider the subject company attri-

butes that make it unique. For starters, the company is a closely held company (and not a company whose shares are listed on a national stock exchange or quoted on the NASDAQ).

The fact that the company is closely held exerts both positive and negative influences when it comes to stock price volatility. On one hand, closely held companies generally (although certainly not

always) are smaller, are less geographically diversified, are less vertically integrated, and have inferior access to capital, compared with their publicly traded peers.

These factors typically cause the required rate of return on an equity investment in a closely held company to be greater than the required rate of return on an equity investment in a publicly traded company. A greater cost of equi-

TABLE 3: DAILY INDEX VALUE CHANGE

Date (2011)	DOW		STAFFING COMPANIES [a]	
	Closing Price	Daily Change (Percent)	Closing Price	Daily Change (Percent)
August 12	11,269.02	+ 1.1	163.97	- 0.1
August 11	11,143.31	+ 3.9	164.20	+ 5.7
August 10	10,719.94	- 4.6	155.40	- 5.3
August 9	11,239.77	+ 3.7	164.12	+ 5.0
August 8	10,809.85	- 5.5	156.36	- 8.3
August 5	11,444.61		170.46	

[a] Based on an index comprised of the selected guideline PTCs identified in Table 2.

ty generally is characterized by greater volatility of returns.

To illustrate this point, let's consider the volatility of two investments: one with a low required rate of return (i.e., long-term U.S. treasury bonds) and one with a high required rate of return (i.e., equity of a start-up biotech company). The volatility of returns for the start-up company typically will be greater than volatility of returns for the debt investment. That is, the investment with the higher required rate of return has the higher expected stock price volatility. Therefore, if (a) the required return on equity is greater for a closely held company than its publicly traded peers, and (b) a higher required return on equity is associated with a higher expected volatility, then the stock price volatility of a closely held company generally should be *higher* than the stock price volatility of its publicly traded peers.

However, there are at least four reasons why this may not necessarily be true:

- Closely held companies typically are owned by a small number of shareholders.

- Shares of closely held company stock typically trade infrequently.
- Closely held companies are not subject to the same disclosure and reporting requirements as PTCs.
- Publicly traded company stock prices often are affected by short term and macroeconomic factors that generally do not exert the same level of influence on the fair market value of closely held company stock.

Those four related factors cause the volatility of closely held companies to be *lower* than the volatility of publicly traded companies.

To illustrate this point, consider the stock of a closely held staffing company (ABC Co.) whose shares are owned by (a) the ABC founder and (2) the company employees through an ESOP. Let's further assume that ABC is appraised annually for ESOP administration purposes. In the annual ESOP appraisal of ABC, the valuator will consider, among other factors:

- The historical, recent, and projected results of ABC Co.
- The state of the industry and the

economy in which ABC operates

- The prices of guideline PTCs
- How the change in value from the last appraisal corresponds to the prior three factors

As a sanity check on the annual stock price change, the valuator may compare, for example, the annual percent change in the ABC Co. share price with the percent change in the ABC aggregate projected cash flow. This comparison may involve the values in the current appraisal and the values in the prior appraisal.

Compare this to the factors that cause the share price of publicly traded company stock to change. The share price of a PTC stock increases or decreases based on many of the same factors that cause the share price of closely held company stock to change. However, a PTC share price also is largely affected by macroeconomic news, short-term factors, and the whims of non-employee investors.

Just look at the results during the week ending August 12, 2011, of (a) the Dow Jones Industrial Average index and (b) a staffing company index that was created using the guideline PTCs identified in Table 2. The closing price of the Dow and a staffing company index during that week is presented in Table 3.

The change in Dow closing price from August 5 to August 12 in Table 3 was a 1.5 percent decrease. The absolute daily change during that period exceeded 3 percent in four out of five days. In one 15-minute period on August 11, the Dow declined by 130 points; on that day the Dow was up nearly 4 percent.

Major news events around these dates contributed to the index value fluctuation: the U.S. economy remained weak and a recession was looming, U.S. debt had recently been downgraded, and the recent debt ceiling fight was trouble-

some to investors. These negative factors explain why stock market investors were pessimistic and/or cautious, but they alone do not explain the significant volatility that occurred during that week.

According to Christopher Blum, the chief investment officer of the U.S. Behavioral Finance Group at J.P. Morgan Asset Management, “Markets have been absolutely in panic mode. Rumors were going around that were wildly fantastic, and the market sells off on it anyway—you can see how irrational behavior is.”⁵

The same dramatic daily swings that occurred in the Dow also occurred in a staffing company index. The staffing company index analyzed in Table 3 is based on the guideline PTCs identified in Table 2. As presented in Table 3, the daily index value change for staffing companies was greater than or equal to 5 percent in four of the five days reviewed.

Let’s take a step back for a moment, consider the examples above, and make some common-sense observations.

First, the collective outlook regarding the companies that comprise the Dow did not decline 5.5 percent Monday, increase 3.7 percent Tuesday, decline 4.6 percent Wednesday, and then increase 3.9 percent on Thursday. Nor did the staffing companies’ outlook change this dramatically from day to day. There were other factors at play besides the changing outlook of the individual companies that comprise the indexes listed on Table 3.

Second, the share price of closely held company stock, like that of the made-up ABC Co., would probably not change the same way that the Dow or the staffing company index changed between August 5 and August 12, 2011.

For example, assume that a hypothetical buyer of ABC Co. equity had spent

months conducting due diligence on ABC and was ready to close the transaction in mid-August 2011. That hypothetical buyer would probably not be willing to pay more for ABC if the transaction closed on either August 9 or August 11 (days when the stock market was up) than he or she would be willing to pay on either August 10 or August 12 (days the stock market was down). Instead, the purchase price would probably be fixed in the days or even the weeks preceding the closing of the transaction. The purchase price in this hypothetical transaction may be subject to certain closing adjustments. However, closing adjustments are typically related to the final target company financial statements, and not to regional or national economic news. Put another way, could a private equity buyer really demand a reduced price for closely held company equity because (for example) the monthly housing starts failed to meet analyst’s expectation?

The above example is meant to explain why a closely held company’s stock price may not be affected by the same short-term, macroeconomic, or behavioral factors (at least not to the same degree) that lead to significant volatility in the stock of PTCs.

These common-sense observations suggest that the stock price of closely held company shares should be less volatile than PTC stock prices.

To reiterate, a higher required rate of return for closely held company equity compared with PTC equity generally supports a *higher* level of volatility for closely held company stock. Conversely, the additional factors that affect the share price of PTC stock—which exert less immediate and less significant impact on closely held company stock—supports a *lower* level of volatility for closely held company stock than PTC stock.

CONCLUSION

After the valuator has analyzed the guideline PTC volatility and estimated industry volatility, how does the valuator relate this estimate to the subject closely held company for use in the BSM option pricing model? The answer is: it depends.

It depends on the judgment of the analyst, the outlook for the subject company, and many other factors. If, for example, a subject company is a one-year-old software provider with no sales and rapid projected growth, then the company stock price volatility may be greater than the selected guideline PTCs. If, on the other hand, the company is 50 years old and operates two dozen grocery stores, then that company’s stock price volatility may be lower than the selected guideline PTCs stock price volatility.

The point of this discussion is not to provide a roadmap or formula that you can follow to estimate stock price volatility for closely held companies using the BSM option pricing model. Instead, the point is to provide the reader with a number of factors that can be considered when making the volatility estimate. By considering the factors presented above, you will reach a more supportable conclusion and produce a more supportable narrative report. VE



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⁵ “Hitting the Panic Button,” *Wall Street Journal MarketBeat*, April 12, 2011, <http://blogs.wsj.com/marketbeat/2011/08/12/hitting-the-panic-button/>.