

*Gift and Estate Tax Valuation Insights*

## THE USE OF THEORETICAL MODELS TO ESTIMATE THE DISCOUNT FOR LACK OF MARKETABILITY

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*Valuations of noncontrolling ownership interests in closely held companies are often performed for estate tax, gift tax, charitable contribution, or other taxation-related purposes. One important procedure in the closely held business valuation analysis is the estimation and application of an appropriate discount for lack of marketability (DLOM). The models commonly used to quantify the DLOM generally fall into one of two categories: (1) empirical models and (2) theoretical models. This discussion focuses on the common theoretical models used by valuation analysts to quantify the DLOM applicable to a noncontrolling ownership interest in a closely held company.*

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### INTRODUCTION

The consideration of the appropriate discount for lack of marketability (DLOM) for a noncontrolling equity interest may be a significant issue in valuations for estate tax, gift tax, or other taxation purposes. The impact of the DLOM on the controlling equity interest valuation may be substantial. In addition, the selection of the subject-specific DLOM may be a controversial aspect of the valuation.

When a DLOM is applicable to the subject interest valuation, analysts often rely on two types of models to quantify the appropriate DLOM:

1. empirical models
2. theoretical models

The principles and applications of the empirical models that are often used to quantify the appropriate DLOM have been discussed in prior issues of *Insights*. Generally, empirical models use analyses that are based on empirical capital market transaction observations—rather than on theoretical economic principles. Alternatively, the theoretical DLOM models do not rely on actual capital market pricing evidence. Rather, the theoretical DLOM models are based on fundamental microeconomic relationships.

The theoretical models used to estimate the DLOM applicable to the valuation of a private company security generally fall into two categories:

1. option pricing models
2. discounted cash flow models

First, this discussion offers insights on the published studies related to the theoretical DLOM models. Second, this discussion provides insights related to the appropriate application of the theoretical DLOM models.

### OPTION PRICING MODELS

Option pricing models assume that the cost to purchase a stock option relates directly to the measurement of the DLOM. Three published DLOM studies that rely on option-pricing theory are summarized below.

#### The Chaffee Study

David B.H. Chaffee, III, authored a 1993 DLOM option pricing study. In this study, Chaffee related the cost to purchase a (European) put option to the DLOM.

Chaffee theorized that, “if one holds restricted or non-marketable stock and purchases an option to sell those shares at the free market price, the holder has, in effect, purchased marketability for those shares. The price of that put is the discount for lack of marketability.”<sup>1</sup>

Chaffee relied on the Black-Scholes option pricing model to estimate the price of the option in his model. The inputs to the Black-Scholes option pricing model are (1) stock price, (2) strike price, (3) time to expiration, (4) interest rate, and (5) volatility.

In the Chaffee model, the stock price and the strike price equal the marketable value of the private company stock as of the valuation date; the time to expiration equals the time the securities are expected to remain nonmarketable; the

interest rate is the cost of capital; and, volatility is a judgmental factor based on the volatility of guideline publicly traded stocks.

According to Chaffee, volatility for small privately owned companies is likely to be 60 percent or greater. Chaffee reached this conclusion based on an analysis of the volatility for small public companies that are traded in the over-the-counter market.

According to the Chaffee study, the appropriate DLOM for a privately held stock with a two-year required holding period and a volatility between 60 percent and 90 percent is between 28 percent and 41 percent.

According to Chaffee, “considering that volatility for shares of most smaller, privately held companies fit the ‘VOL 60%-70%-80%-90%’ curves, a range of put prices of approximately 28% to 41% of the marketable price is shown at the two-year intercept. At the four-year intercept, these ranges are 32% to 49%, after which time increases do not substantially change the put price.”

Chaffee noted that his findings are downward biased (due to the reliance on European options in his model). Therefore, Chaffee concluded that his findings should be viewed as a minimum applicable DLOM.

### The Longstaff Study

Francis A. Longstaff also authored a study that relies on stock options to estimate the DLOM for the valuation of private company stock.<sup>2</sup> Whereas the Chaffee study is based on avoiding losses, the Longstaff study is based on unrealized gains. Another difference from the Chaffee study is that the Longstaff study (allegedly) provides an estimate for the upper bound on the value for marketability.

The Longstaff study is based on the price of a “look-back” option. The Longstaff study assumes an investor has (1) a single-security portfolio, (2) perfect market timing, and (3) trading restrictions that prevent the security from being sold at the optimal time. The value of marketability, based on these assumptions, is the payoff from an option on the maximum value of the security, where the strike price of the option is stochastic.

The results from the Longstaff study are summarized in Table 1 below.

As Table 1 illustrates, for a 5-year holding period and 30 percent standard deviation, the appropriate DLOM is over 65 percent. Longstaff analyzed securities with a volatility between 10 percent and 30 percent because, “This range of volatility is consistent with typical stock return volatilities.”

However, as noted above, small stocks (such as those traded over-the-counter and analyzed by Chaffee) typically have greater volatility, all else equal.

According to Longstaff:

This analysis provides a number of new insights about how marketability restrictions affect security values. First, we show that discounts for lack of marketability can be large even when the length of the marketability restriction is very short. Second, the upper bound provides a benchmark for estimating the valuation effects of marketability restrictions such as circuit breakers, trading halts, and prohibitions on program trading. Finally, these results allow us to assess directly whether empirical estimates of discounts for lack of marketability are consistent with rational market pricing.<sup>3</sup>

**Table 1**  
**The Longstaff Study**  
**Upper Bounds for the DLOM Adjustment**

Marketability Restriction Period	Standard Deviation = 10%	Standard Deviation = 20%	Standard Deviation = 30%
1 day	0.421	0.844	1.268
5 days	0.944	1.894	2.852
10 days	1.337	2.688	4.052
20 days	1.894	3.817	5.768
30 days	2.324	4.691	7.100
60 days	3.299	6.683	10.153
90 days	4.052	8.232	12.542
180 days	5.768	11.793	18.082
1 year	8.232	16.984	26.276
2 years	11.793	24.643	38.605
5 years	19.128	40.979	65.772

## The Finnerty Study

John D. Finnerty conducted an option-pricing study that, “tests the relative importance of transfer restrictions on the one hand and information and equity ownership concentration effects on the other in explaining private placement discounts.”<sup>4</sup> The Finnerty option-pricing study is an extension of the Longstaff study. Unlike Longstaff, however, Finnerty did not assume that investors have perfect market timing ability. Instead, Finnerty modeled the DLOM as the value of an average strike put option.

In addition to analyzing stock-options, Finnerty analyzed 101 private placements of restricted stock that occurred between January 1, 1997, and February 3, 1997.

The Finnerty private placement study concluded price discounts of 20.13 percent and 18.41 percent for the day prior to the private placement and for 10 days prior to the private placement, respectively.

With regard to his option-pricing model, Finnerty concluded:

The model (5)-(6) calculates transferability discounts that are consistent with the range of discounts observed empirically in letter-stock private placements for common stocks with volatilities between  $\delta = 30$  percent and  $\delta = 70$  percent but the implied discounts are greater than (less than) those predicted by the model for lower (higher) volatilities.

In addition, Finnerty made the following observation about the importance of dividends, volatility, and the DLOM:

My model implies that when the stock price volatility is under 30 percent, the appropriate discount is smaller than the customary discount range of about 25 percent to 35 percent. For example, when  $\delta$  is between 20 percent and 30 percent and there is a two-year restriction period, the proper discount is in the range from 15.76 percent to 20.12 percent for a non-dividend-paying stock and in the range from 11.50 percent to 15.96 percent for a stock yielding 3.0 percent. The halving of the initial restriction period under Rule 144 since February 1997 has roughly halved the transferability discount.<sup>5</sup>

Finally, Finnerty proposed an explanation for the small price discounts observed in private placement studies conducted by (1) Karen Wruck<sup>6</sup> and (2) Hertz and Smith,<sup>7</sup> where the observed DLOM was less than 15.0 percent:

The difference is due in part to the information and ownership concentration effects that accompany a common stock private placement, but may also be

due to mispricing of the forgone put option. In any case, the discount varies directly with the stock's volatility.

## Long-Term Equity Anticipation Securities (LEAPS) Studies

In September 2003, Robert Trout published a LEAPS study.<sup>8</sup> In June 2005, Ronald Seaman updated the Trout LEAPS study.<sup>9</sup> In Winter 2005, Seaman expanded his earlier study.<sup>10</sup>

Each of these LEAPS studies was conducted with similar research logic and with similar research procedures. This discussion concurrently reviews these three LEAPS studies.

A LEAP, generally, is a long-term put option. LEAPS offer price protection for up to two years in the future. Therefore, an investor who desires protection against stock price declines can purchase a LEAP put option. The LEAPS studies examined the cost of buying LEAP puts. The cost of the LEAP put option divided by the stock price serves as the basis for the DLOM.

Trout examined nine LEAPS as of March 2003 (with options expiring January 2005). The nine LEAPS examined were for large companies with actively traded securities.<sup>11</sup>

According to the Trout study, “The data concerning the relative cost of puts as an insurance premium cost equal to about 24 percent of price. This finding suggests that the minimum discount that one should assign for the lack of marketability of holding privately held stock is at least 24 percent.”

The Seaman study updated and extended the Trout study up through June 2005. The Seaman study determined if holding period and risk affected the LEAPS cost (i.e., the price discount). This first Seaman study considered 100 randomly selected securities where LEAP options traded.

The results of the Seaman study are summarized in Table 2.

**Table 2**  
**The Seaman Study**

Safety rank:	1	2	3	4	5
Beta					
Average	0.82	0.95	1.10	1.55	1.87
Median	0.80	0.95	1.10	1.55	1.88
One-Year Price Discount					
Average	7.0%	8.1%	10.6%	15.5%	20.0%
Median	8.3%	7.5%	9.2%	13.8%	17.0%
Two-Year Price Discount					
Average	9.4%	11.1%	14.9%	20.3%	30.3%
Median	9.3%	10.4%	13.8%	18.7%	31.0%

The safety rank measure above is based on the *ValueLine Investment Survey*. A measure of 1 implies less risk and a measure of 5 implies greater risk. As demonstrated above, the cost to hedge using LEAP put options increases as the holding period increases. In addition, it is more expensive to hedge as a company's risk increases.

Seaman noted that this evidence, "serves primarily as a sanity check on the size of discount for lack of marketability we choose for a particular company."<sup>12</sup>

Seaman's second article, also published in 2005, expanded his first article to include the effect size has on the discount. Seaman expanded the number of companies analyzed from 100 to 261. Seaman concluded that, "as company size decreases (in either revenues or total assets), the discount required increases."<sup>13</sup>

The authors of the three LEAPS studies concluded that the observed DLoms are appropriately viewed as benchmark minimum price discounts when applied to privately held companies. The discounts should represent minimum price discounts because: (1) the market value of the underlying securities on which the LEAPS are based is often much larger than the value of the privately held subject company, (2) the underlying securities on which the LEAPS are based are marketable, (3) the LEAPS themselves can be sold at any time during the holding period, and (4) there is a known liquidity event for the LEAPS.

### General Option Pricing Model Conclusions

Generally, the option pricing studies presented above conclude similar price discounts to the empirical studies. In the Chaffee, Longstaff, and Finnerty studies, the appropriate DLOM for a privately held company (given certain volatility assumptions) reaches 65 percent. In the LEAPS studies, the price discount is much lower, but it is reported to represent a minimum DLOM.

Because of their nature, option pricing studies consider only certain aspects of closely held companies. That is, the option pricing studies generally only consider the factors that affect option pricing the most: holding period and volatility. Although other factors are present in option pricing, the holding period and volatility factors have the greatest impact on option pricing.

Therefore, option pricing studies may understate the DLOM. This is because these studies ignore other factors that may reduce the marketability for a privately held company (e.g., contractual transferability restrictions).

Intuitively, basing the size of the DLOM on these two factors makes sense. The holding period is discussed in detail in the restricted stock studies. As the restricted stock studies indicate, the longer the required holding period, the greater price concession (i.e., DLOM) a buyer expects to receive. As an example, when the SEC Rule 144 hold-

ing period was reduced from 2 years to 1 year, the average restricted stock price discount declined.

This evidence, combined with the option-pricing studies, illustrates that the expected required holding period has a direct impact on the appropriate DLOM for a privately held company. As the expected required holding period increases, so should the DLOM, holding all other factors constant.

Also, volatility is directly related to the magnitude of the DLOM. When an investor owns a security that is restricted from trading, the investor assumes the risks (among other risks) of (1) not being able to sell the investment if the value begins to suddenly decline and (2) not being able to sell the investment to reallocate funds to another investment. The first risk factor is materially affected by highly volatile stocks. As volatility increases, the risk of significant stock price depreciation increases. Therefore, as volatility increases, the risk related to holding a nonmarketable security likewise increases.

### The Application of Option Pricing Models

The option pricing model studies provide a general methodology for analyzing the DLOM. These theoretical option pricing model studies make several contributions to the empirical DLOM research.

First, the option pricing studies indicate that, for stocks with low volatility, the appropriate DLOM may be below the range of average price discounts reported in the empirical studies. The practical problem, of course, is how to determine the volatility for the stock of a privately owned company.

The Chaffee study assumed that the appropriate volatility for most privately held companies is likely in excess of 60 percent, based on the average volatility of small publicly traded company stocks. Alternatively, Longstaff analyzed price discounts for companies with volatility ranging between 10 percent and 30 percent, based on publicly traded companies of all sizes.

When using these data, it is important for the valuation analyst to consider whether the subject company qualitative and quantitative factors warrant a low estimated volatility. If so, these factors may support the use of a lower than average DLOM.

Alternatively, if the subject company would warrant a volatility of 60 percent or greater, then the appropriate DLOM may be above the average DLOM indicated in the various DLOM studies (all other factors being equal).

Second, the option pricing studies show how the length of the required holding period has a material effect on the magnitude of the DLOM. As the Longstaff study showed, as the holding period increases from 1 year, to 2 years, to 5 years, the DLOM for a stock with a 20 percent volatility increases from 17 percent, to 25 percent, to 41 percent.

The Chaffee and Finnerty studies reached similar conclusions regarding the required holding period and the DLOM. If, due to contractual restrictions, a limited pool of potential buyers, or other factors, the subject company stock is not expected to become marketable for many years, then the use of a greater than average discount may be appropriate. However, if the subject company is contemplating a liquidity event such as IPO, merger, or sale, then the DLOM may be lower than the average price discount indicated by the empirical studies.

The usefulness of these option pricing studies is mitigated by at least two factors.

First, an important assumption of each of the above-referenced option pricing studies is a liquidity event. The reality for an owner of private company stock is that he or she may never experience a liquidity event. This assumption would warrant a price discount greater than what is indicated by the option pricing studies.

Second, an owner of private company stock does not have the ability to hedge his or her investment in the options market. Stock options on small, thinly traded companies rarely exist, and the market for private company stock or options on that stock simply does not exist.

If the implied DLOM from a particular option pricing model is 30 percent (when the strategy is actually available to investors), then the implied DLOM for shares of private company stock would be even greater (when the strategy does not actually exist).

## DISCOUNTED CASH FLOW MODELS

A second category of theoretical studies is based on the discounted cash flow method. The discounted cash flow method is based on the principle that value equals the present value of future income.

Z. Christopher Mercer and Travis W. Harms described how the discounted cash flow model relates to the DLOM: “Quantitative analyses therefore estimates the value of illiquid interests based on the expectation of benefits (distributions or dividends and proceeds of ultimate sales) over relevant expected holding periods using appropriate discount rates to equate with present values. The process of doing this analysis, in the context of valuing a business at the marketable minority interest level, determines the applicable marketability discount.”<sup>14</sup>

Two commonly referenced studies that rely on the discounted cash flow methodology are summarized below.

### The Quantitative Marketability Discount Model (QMDM)

The QMDM is a shareholder-level discounted cash flow model that uses a quantitative analysis to precisely

calculate the DLOM. The QMDM was developed by Z. Christopher Mercer.

The QMDM calculates the DLOM based on:

1. the expected growth rate in the subject company value,
2. the expected interim cash flow,
3. the expected holding period, and
4. the required holding period return.

In his book, *Quantifying Marketability Discounts*,<sup>15</sup> Mercer provides detailed guidance with regard to estimating these four factors.

In the application of the QMDM, an analyst first values the subject company at the entity level, resulting in a stock valuation as if the stock was readily marketable. Next, the shareholder value is calculated. The shareholder value represents the nonmarketable value of the subject stock.

To calculate the shareholder value, the valuation analyst increases the value of the subject company by the growth rate during the expected holding period. The valuation analyst next discounts the future company value using the required holding period return. The valuation analyst then adds the present value of the dividend stream received during the holding period to this present value.

The resulting value equals the shareholder value. The calculation of one minus the ratio of shareholder value to enterprise value equals the DLOM, based on the QMDM.

While intuitively appealing, the QMDM has practical limitations. For example, the DLOM computed using the model is highly subject to the model inputs. In the *Estate of Weinberg v. Commissioner*, the Tax Court noted, “slight variations in the assumptions used in the model produce dramatic differences in the results.”<sup>16</sup>

In the *Estate of Janda v. Commissioner*, the Tax Court was concerned with the magnitude of the DLOM calculated using the QMDM model. The Tax Court noted, “We have grave doubts about the reliability of the QMDM model to produce reasonable discounts, given the generated discount of over 65%.”<sup>17</sup>

### The Tabak Model

The Tabak model is another discounted cash flow model used to estimate the appropriate DLOM for a privately owned company based on the capital asset pricing model (CAPM). The Tabak model presents a unique way to estimate the DLOM.

This is because the Tabak model “focuses on the extra risks imposed on the owner of a security or interest in a business enterprise, and not on the lack of access to capital. In brief, the theory uses market data on the additional

return that investors require in order to hold a risky asset, measured by the equity risk premium, to extrapolate the extra return that the holder of an illiquid asset would require.”<sup>18</sup>

### General Discounted Cash Flow Model Conclusions

The discounted cash flow models discussed above provide an interesting analysis regarding (1) the cause of the DLOM and (2) the measurement of the DLOM. Although the discounted cash flow models are controversial, they are based on generally accepted financial theory.

For example, calculating the present value of the cash flow received at the “shareholder level” during the expected holding period, as in the QMDM, is a theoretically correct process. However, the model results are highly sensitive to the model inputs.

In addition, the model inputs used in the QMDM and the Tabak model require the application of analyst’s judgment—that is, a subjective factor that the models attempt to overcome. Finally, the QMDM, and especially the Tabak model, have not been widely accepted by valuation analysts or by the federal courts.

Because of these factors, valuation analysts should only rely on the discounted cash flow models presented above when the inputs can be measured with relative certainty. In addition, an analysis of the empirical DLOM models should typically accompany the use of the discounted cash flow DLOM models.

### SUMMARY AND CONCLUSION

The preceding discussion offered insights into some of the theoretical models that are often relied on to estimate the DLOM. It is important to remember that an asset is not simply either marketable or nonmarketable. Rather, there are varying degrees of marketability.

The extent to which the theoretical models presented measure this degree of marketability depends largely on the quality and precision of the inputs used. These factors are particularly important when the valuation analyst uses the discounted cash flow models.

While the studies discussed above describe a starting point to estimate the DLOM using theoretical models, the facts and circumstances of each subject valuation will determine the appropriate DLOM. It is a matter of valuation analyst judgment to select a DLOM based on:

1. the empirical DLOM evidence,
2. the theoretical DLOM evidence, and
3. the facts and circumstances of each individual case.

### Notes:

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2. Francis A. Longstaff. “How Much Can Marketability Affect Security Values?” *The Journal of Finance*, December 1995, pp. 1767–74.
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8. Robert R. Trout, “Minimum Marketability Discounts,” *Business Valuation Review*, September 2003, pp. 124–6.
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14. Z. Christopher Mercer and Travis W. Harms, “Marketability Discount Analysis at a Fork in the Road,” *Business Valuation Review*, December 2001, p. 23.
15. Z. Christopher Mercer, *Quantifying Marketability Discounts* (Memphis: Peabody Publishing, 1997).
16. *Estate of Weinberg v. Commissioner*, T.C. Memo 2000-51 (February 15, 2000).
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